7. Goniometrické nerovnice

Příklady

PŘI ŘEŠENÍ VYUŽÍVÁME
- GRAFŮ GON. FCÍ
- JEDNOTKOVOU KRUŽNICI

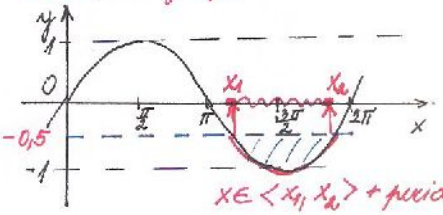
① Řeším v $\mathbb{R}$

a) $\sin x \leqq -0{,}5$

$y \leqq -0{,}5$

$(y = \sin x)$

1. způs. z grafu



$x \in \langle x_1, x_2 \rangle$ + perioda

- musíme určit meze x_1, x_2

[=> řešíme rce $\sin x = -0{,}5$

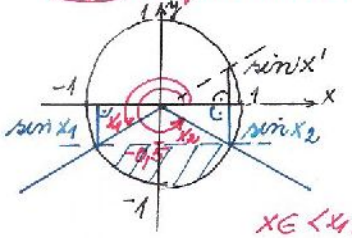
[$\ominus$ III., IV. kv. $x' = 30° = \frac{\pi}{6}$]

III. kv.
$x_1 = \pi + x' + 2k\pi$
$x_1 = \pi + \frac{\pi}{6} + 2k\pi$
$x_1 = \frac{7\pi}{6} + 2k\pi$

IV. kv.
$x_2 = 2\pi - x' + 2k\pi$
$x_2 = 2\pi - \frac{\pi}{6} + 2k\pi$
$x_2 = \frac{11\pi}{6} + 2k\pi$

$\mathcal{K} = \bigcup_{k \in Z} \left\langle \frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi \right\rangle$

2. způs. z jednotkové kružnice



! POZOR na označování $\sin x$ [y-ová souř.] viz čl. 4

$x \in \langle x_1, x_2 \rangle$ + perioda

- musíme určit meze x_1, x_2

[=> kořeny rce $\sin x = -0{,}5$

[$x' = 30° = \frac{\pi}{6}$ $\ominus$ III., IV. kv.]

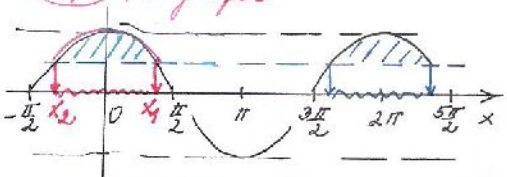
III. kv.
$x_1 = \pi + \frac{\pi}{6} + 2k\pi$
$x_1 = \frac{7\pi}{6} + 2k\pi$

IV. kv.
$x_2 = 2\pi - \frac{\pi}{6} + 2k\pi$
$x_2 = \frac{11\pi}{6} + 2k\pi$

$\mathcal{K} = \bigcup_{k \in Z} \left\langle \frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi \right\rangle$

b) $\cos x > \frac{1}{2}$

1. způs. z grafu



$x \in (x_2, x_1)$

meze: pro jejich určení beremě x_1, x_2 tak, aby úsek byl souvislý

[$\cos x = \frac{1}{2}$

[$x' = 60° = \frac{\pi}{3}$]

I. kv. $x_1 = x' + 2k\pi = \frac{\pi}{3} + 2k\pi$

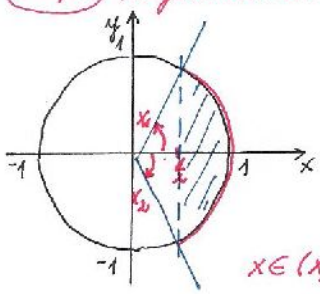
IV. kv. - lépe např. úhel (aby souvr. úsek)

$x_2 = 2\pi - x' + 2k\pi = \frac{3\pi}{2} + 2k\pi \Rightarrow$

$\Rightarrow x_2 = -x' + 2k\pi = -\frac{\pi}{3} + 2k\pi$

$\mathcal{K} = \bigcup_{k \in Z} \left(-\frac{\pi}{3} + 2k\pi, \frac{\pi}{3} + 2k\pi \right)$

2. způs. z jednotkové kružnice



! POZOR na označ. cos x [x-ová souř.] viz čl. 4

$x \in (x_2, x_1)$

meze x_1, x_2

[$\cos x = \frac{1}{2} \Rightarrow x' = 60° = \frac{\pi}{3}$]

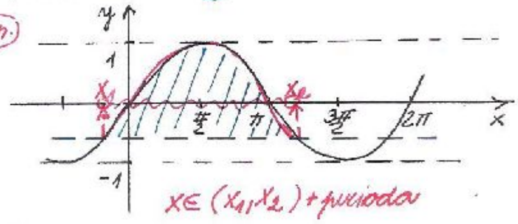
$x_1 = \frac{\pi}{3} + 2k\pi$

$x_2 = -\frac{\pi}{3} + 2k\pi$

$\mathcal{K} = \bigcup_{k \in Z} \left(-\frac{\pi}{3} + 2k\pi, \frac{\pi}{3} + 2k\pi \right)$

c) $\sin x > -\frac{\sqrt{3}}{2}$

1. zp.



$x \in (x_1, x_2)$ + perioda

určení mezí

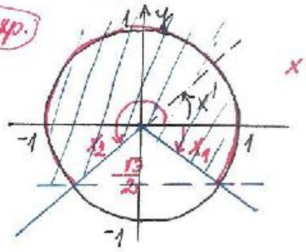
$\left[\sin x' = \frac{\sqrt{3}}{2} \Rightarrow x' = 60° = \frac{\pi}{3}\right]$

$x_1 = -x' + 2k\pi = -\frac{\pi}{3} + 2k\pi$

$x_2 = \pi + x' + 2k\pi = \pi + \frac{\pi}{3} + 2k\pi = \frac{4\pi}{3} + 2k\pi$ (III. kv.)

$K = \bigcup_{k\in Z} \left(-\frac{\pi}{3} + 2k\pi,\ \frac{4\pi}{3} + 2k\pi\right)$

2. zp.



$x \in (x_1, x_2)$ + perioda

meze: $\left[x' = 60° = \frac{\pi}{3}\right]$

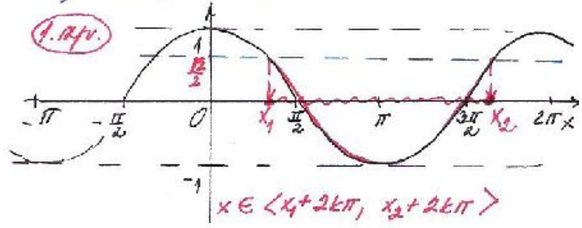
$x_1 = -\frac{\pi}{3} + 2k\pi$

$x_2 = \pi + \frac{\pi}{3} + 2k\pi = \frac{4\pi}{3} + 2k\pi$

$K = \bigcup_{k\in Z} \left(-\frac{\pi}{3} + 2k\pi,\ \frac{4\pi}{3} + 2k\pi\right)$

d) $\cos x \leqq \frac{\sqrt{2}}{2}$

1. zp.



$x \in \langle x_1 + 2k\pi,\ x_2 + 2k\pi \rangle$

určení mezí

$\left[\cos x' = \frac{\sqrt{2}}{2} \Rightarrow x' = 45° = \frac{\pi}{4}\right]$

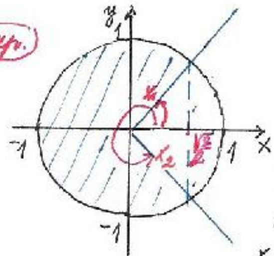
$x_1 = x' + 2k\pi = \frac{\pi}{4} + 2k\pi$

$x_2 = 2\pi - x' + 2k\pi = 2\pi - \frac{\pi}{4} + 2k\pi$ (IV. kv.)

$x_2 = \frac{7\pi}{4} + 2k\pi$

$K = \bigcup_{k\in Z} \left\langle \frac{\pi}{4} + 2k\pi,\ \frac{7\pi}{4} + 2k\pi \right\rangle$

2. zp.



POZOR! – cos na ose x (x-ová souř.)

$x \in \langle x_1, x_2 \rangle$ + perioda

meze: $[x' = 45°]$

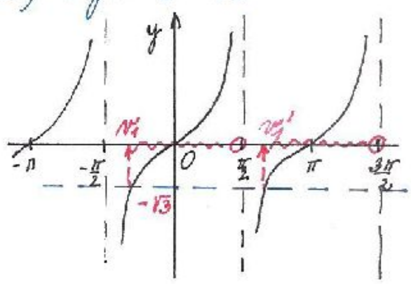
$x_1 = \frac{\pi}{4} + 2k\pi$

$x_2 = 2\pi - x' = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} + 2k\pi$

$K = \bigcup_{k\in Z} \left\langle \frac{\pi}{4} + 2k\pi,\ \frac{7\pi}{4} + 2k\pi \right\rangle$

② Řešte v R

a) $\operatorname{tg} x \geqq -\sqrt{3}$



– hledáme souvislé úseky

$x \in \langle x_1, \frac{\pi}{2})$ + perioda

nebo $[\operatorname{tg} \to {}^{+}k\pi]$

$x \in \langle x_1', \frac{3\pi}{2})$ + perioda

meze $\left[\operatorname{tg} x' = \sqrt{3} \Rightarrow x' = 60° = \frac{\pi}{3}\right]$

$x_1 = -x' + k\pi = -\frac{\pi}{3} + k\pi$

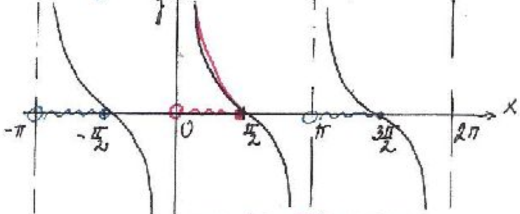
$\left(x_1' = \pi - x' + k\pi = \frac{2\pi}{3} + k\pi\right)$ ⊖ II. kv.

POZNÁMKA: – z jednotkové kružnice není tak dobře vidět

$K = \bigcup_{k\in Z} \left\langle -\frac{\pi}{3} + k\pi,\ \frac{\pi}{2} + k\pi \right)$

nebo $K = \bigcup_{k\in Z} \left\langle \frac{2\pi}{3} + k\pi,\ \frac{3\pi}{2} + k\pi \right)$

b) $\operatorname{cotg} x \geqq 0$

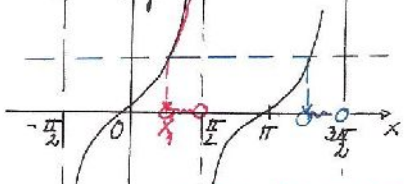


$x \in (0, \frac{\pi}{2}\rangle$ + per.

$(0 + k\pi, \frac{\pi}{2} + k\pi\rangle$

$K = \bigcup_{k\in Z} \left(k\pi,\ \frac{\pi}{2} + k\pi\right\rangle$

c) $\operatorname{tg} x > 1$



meze: $\left[\operatorname{tg} x' = 1,\ x' = 45°,\ x' = \frac{\pi}{4}\right]$

$x_1 = x' + k\pi$

$x_1 = \frac{\pi}{4} + k\pi$

$x \in (x_1, \frac{\pi}{2})$ + perioda

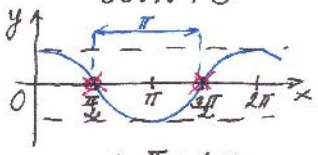
$K = \bigcup_{k\in Z} \left(\frac{\pi}{4} + k\pi,\ \frac{\pi}{2} + k\pi\right)$

③ Určete definiční obory funkcí

a) $y = \frac{1}{\cos x}$

[jmen. zlomku $\neq 0$]

$\cos x \neq 0$



$x \neq \frac{\pi}{2} + k\pi$

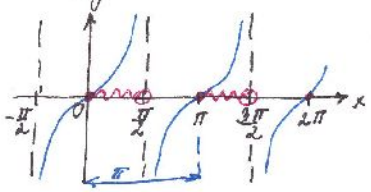
$x \neq \frac{\pi}{2}(1+2k)$

$\mathcal{D} = R - \bigcup_{k \in Z} \{\frac{\pi}{2}(2k+1)\}$

b) $y = \sqrt{\operatorname{sg} x}$

[pod sudou odmoc. výraz ≥ 0]

$\operatorname{sg} x \geq 0$



$x \in \langle 0, \frac{\pi}{2}) +$ perioda

$\mathcal{D} = \bigcup_{k \in Z} \langle 0 + k\pi, \frac{\pi}{2} + k\pi)$

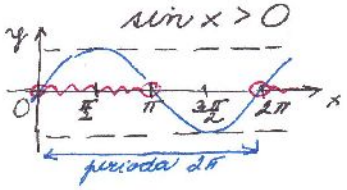
$\left[\mathcal{D} = \bigcup_{k \in Z} \langle k\pi, \frac{\pi}{2} + k\pi)\right]$

c) $y = \frac{1}{\sqrt{\sin x}}$

[prien. zlomku $\neq 0 \wedge$ výraz pod odmocninou ≥ 0]

$\sin x \geqq 0 \wedge \sin x \neq 0$

$\sin x > 0$



$\mathcal{D} = \bigcup_{k \in Z} (0 + 2k\pi, \pi + 2k\pi)$

$\left[\mathcal{D} = \bigcup_{k \in Z} (2k\pi, \pi + 2k\pi)\right]$